202: Principles of electrical science  
**Handout 2: Mathematical principles**

**Learning outcome**

The learner will:

1. Understand mathematical principles which are appropriate to electrical installation, maintenance and design work.

**Assessment criteria**

The learner can:

* 1. identify and apply appropriate **mathematical principles** which are relevant to electrical work tasks.

**Range**

**Mathematical principles**: Fractions and percentages, Algebra, Indices, Transposition, Triangles and trigonometry, Statistics.

**Mathematical principles**

One of the issues with working with electricity is that under normal circumstances we can’t see it, hear it, or see it although we can smell it if something goes wrong. If we are to understand the quantities involved we need to measure the relevant electrical quantities and from these calculate other quantities. So, if we are to understand electrical principles we need to have a good understanding of certain mathematical principles.

**Fractions and percentages**

A fraction represents a part of a whole. When spoken in everyday English, a fraction describes how many parts of a certain size there are, for example, one‑half or 0.5, eight‑fifths or 1.6, three‑quarters or 0.75.

Fractions can be classified in two ways:

* Vulgar fractions
* Decimal fractions

**Vulgar fractions**: A vulgar fraction consists of an integer (whole number) **numerator** displayed above a line (or before a slash), and a non-zero integer **denominator**, displayed below (or after) that line. Some examples are:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |

The number at the bottom (or to the right of the slash) is the denominator and tells us how many pieces an item is divided up by. The number on the top (or to the left of the slash) tells us how many of those pieces we have. For example:

|  |  |  |  |
| --- | --- | --- | --- |
| * The picture to the right represents a cake. * It has been divided into eight equal pieces. * Because the cake has eight equal pieces the **denominator** will be **8**. * We are taking the green pieces of the cake and there are three of these. * These **3** pieces will be the **numerator** or the number of pieces we have. * The resulting fraction will be: | | |  |
|  | or |  |

Fractions with the same denominator are referred to as **like** fractions. Add or subtract the numerators and write the answer as the new numerator above common denominator. For example:

|  |  |  |
| --- | --- | --- |
|  | or |  |

Note that in the second example the answer was but this can be simplified to by cancelling.

If the fractions have different denominators we must first make the fractions equivalent by creating the same denominator. For example:

To solve this, we first had to find the lowest common multiple (**LCM**) which is the smallest number that all denominators will divide into without any remainder in this case, 24. Next, the numerators need to be converted. Divide the original denominator in each case into the LCM and then multiply the respective numerator by the answer; 8 divided into 24 equals 3 times 3 equals 9 and 12 divided into 24 equals 2 times 3 equals 6. We then add the new numerators together to give 15 which is then placed over the new denominator (24) to give . This can then be simplified by cancelling to give .

**Decimal fractions**: A decimal fraction is a fraction where the denominator (the bottom number) is a power of ten (such as 10, 100, 1000, etc).

You can write decimal fractions with a decimal point (and no denominator), which make it easier to do calculations like addition and multiplication on fractions. for example:

Decimal fractions are ideal for use with calculators they can be entered directly into the calculator. Vulgar fractions can be converted to decimal fractions by dividing the numerator by the denominator. For example:

**Percentages**: ‘Percent’ means **out of 100**. The current basic income tax rate is 20 per cent taxable pay, this means that for £100 you will have to pay £20 in income tax. The symbol % means per cent.

How to calculate a percentage.

A percentage is a fraction with a denominator of 100.

60% (60 in each 100) as a fraction is 60/100

60% as a decimal is 0.6.

You will frequently need to find a percentage of a quantity. First, write the percentage as a fraction or a decimal, then multiply by the quantity.

Example: The maximum permitted voltage drop in a lighting circuit is 3% of the supply voltage. Calculate the maximum voltage drop if the supply voltage is 230 volts?

First, write 3% as a fraction:

Now multiply by the quantity:

**Algebra**

Algebra uses letters (like x or y) or other symbols in place of values and is used to find unknown values, with rules for manipulating these symbols. For example, in electrical principles we can calculate the current flowing in a circuit if we know the applied voltage and the resistance of the circuit using the formula:

|  |  |  |  |
| --- | --- | --- | --- |
| Where: | I | = | Current in amperes |
|  | V | = | Voltage in volts |
|  | R | = | Resistance in ohms |

Example: Calculate the current that will flow in a circuit of resistance of 460 ohms when a voltage of 230 volts is applied.

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |

But what if we know, say, the current and resistance but not the voltage? We can still solve the equation by using **transposition** and this is explained in the next paragraph.

**Transposition**

This is also known as ‘**changing the subject of the formulae**’.

|  |  |
| --- | --- |
| transposition.png | In the example on the left, *‘***I***’* is the subject of the formula. By inserting the values for ‘**V**’ and ‘**R**’, the value of the subject ‘**I**’ can be calculated.  If we need to find ‘**V**’, for example, we must **transpose** the formula to make ‘**V**’ the subject.  There is one fundamental rule for transposing a formula, as found below. |

|  |
| --- |
| **Whatever you do to one side of the formula, you must do the same to the other side** |

In other words:

* add the same quantity to both sides of the formula
* subtract the same quantity from both sides of the formula
* multiply both sides of the formula by the same quantity
* divide both sides of the formula by the same quantity
* take ‘functions’ of both sides of the formula; for example, square both sides or ﬁnd the reciprocal of both sides.

Example

I = V/R – make V the subject of the formula.

|  |  |  |  |
| --- | --- | --- | --- |
| I | = |  | (I is currently the subject of the formula.) |

The question is: ‘Make V the subject of the formula’. This means that ‘V’ must be put on its own on one side of the equals sign and the other terms must be on the other side.

|  |  |  |  |
| --- | --- | --- | --- |
| In order to do this, first multiply both sides by R. |  | = |  |
| Now cancel through: |  | = |  |
|  |  | = |  |
| Now reverse the formula: |  | **=** |  |

Example

I = V/R – make R the subject of the formula.

|  |  |  |  |
| --- | --- | --- | --- |
| I | = |  | (I is currently the subject of the formula.) |

The question is: ‘Make R the subject of the formula’. This means that ‘R’ must be put on its own on one side of the equals sign and the other terms must be on the other side.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| In order to do this, first multiply both sides by R. |  | = |  | |
| Now cancel through: |  | = |  | |
|  |  | = |  | |
| Now divide both sides by I: |  | = |  | |
| Now cancel through : |  | = |  | |
|  |  | **=** |  | |
| The formula used in examples 1 and 2 is the **Ohm’s law** formula and is used to calculate the relationship between resistance (R), current (I) and voltage (V).  In formulae (the plural of formula) where there are three values, with two divided or multiplied by each other, the values can be put into a triangle that will allow you to determine easily which variation of the formula should be used. | | | | ohms law triangle.png | |

If we need to find a value, simply cover it and what’s left gives the formula. If the two values are side by side, multiply them; if the two values are one on top of the other then divide the bottom one into the top one to give the following:

|  |  |  |
| --- | --- | --- |
|  |  |  |

**Example 3**

E = B x l x v – make v the subject of the formula.

|  |  |  |  |
| --- | --- | --- | --- |
|  | = |  | (E is currently the subject of the formula.) |

The question is: ‘Make v the subject of the formula’. This means that ‘v’ must be put on its own on one side of the equals sign and the other terms must be on the other side.

In order to do this, first divide both sides by B x l:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | | = |  |  |
| Now cancel through: |  | = |  |  |
|  |  | = |  |  |
| Reverse the formula: |  | **=** |  |  |

**Indices**

A knowledge of powers, or indices as they are often called, is essential for an understanding of most algebraic processes.

Basically, they are a shorthand way of writing multiplications of the same number.

So, suppose we have

We write this as “4 to the power 3”:

43

So

The superscripted number 3 is called the power or index. Note that the plural of index is indices.

Indices can be positive or negative and we generally use them in electrical science to express very large or very small numbers easily.

For example, a typical microwave motion detector uses a frequency of **2 420 000 000Hz**. If you were to have to write numbers of this magnitude or input them into a calculator regularly it would be quite a chore. By using indices, the number could be written as:

**2.42 x 109 Hz**

Effectively, the **109** means that the decimal place, which is initially at the extreme right of the number, is moved 9 places to the **left**.

Another example is that the resistivity of copper is **0.000 000 017 2 ohm/metre3**. Again, to write and input these numbers would be quite a chore. By using negative indices, the number could be written as:

**17.2 10-9 ohm/metre3**

Effectively, the **10-9** means that the decimal place, which is initially at the left end of the number, is moved 9 places to the **right**.

**Standard Form or Scientific Notation**: A number written with one digit to the left of the decimal point and multiplied by 10 raised to some power is written in standard form or with scientific notation. For example:

**43712 = 4.3712 x 104**

**0.036 = 3.6 x 10-2**

**Engineering Notation**: is like scientific notation except that the power of ten is always a multiple of 3. For example:

**43712 = 43.712x 103 = 0.043712 x 106**

**0.036 = 36 x 10-3 = 36000 x 10-6**

In electrical installation, we generally use engineering notation.

**Triangles**

When carrying out various calculations in electrical science we regularly make use of triangles, for example, to calculate impedance, power and power factor. To enable us to do these calculations we need to have a good understanding of **Pythagoras’ Theorem** and **trigonometry**.

|  |  |
| --- | --- |
| **Pythagoras' theorem** states that for all right-angled triangles, ***‘The square on the hypotenuse is equal to the sum of the squares on the other two sides’***.  The hypotenuse is the longest side and it's always opposite the right angle.  If we draw a square on each side of a right‑angled triangle, Pythagoras found that the combined areas of sides A and B (see the diagram to the right) will equal the area of the square on the hypotenuse, side C in this case.  This gives the relationship: |  |

To calculate side C (the hypotenuse) the relationship would become the following equation:

To use Pythagoras’ Theorem we need to know the lengths of two of the sides to find the length of the third. The above equation can be transposed to enable us to find the other two sides as follows:

We use Pythagoras’ Theorem to calculate power and impedance in a.c. circuits and you will be shown how to apply the Theorem later in your studies.

**Trigonometry**: is a branch of mathematics that studies relationships involving lengths and angles of triangles. Again, relating to right‑angle triangles, if we know the length of one side and one angle we can calculate all the others.

The ratios of the sides of a right triangle are called trigonometric ratios. Three common trigonometric ratios are the **sine** (sin), **cosine** (cos), and **tangent** (tan).

These are defined for acute angle A below:

|  |  |
| --- | --- |
|  |  |

We use trigonometry to calculate power factor in a.c. circuits and you will be shown how to apply the this later in your studies.

**Statistics**

This is a branch of mathematics dealing with the collection, analysis, interpretation, presentation, and organization of data. We would normal start with data collection which could be, for example, the number of people with red cars, blue cars, silver cars or white cars; this is referred to as the **population**.

Once the data has been collected it can then be analysed using simple statistical tools including ‘**range**’, ‘**average**’ (‘**mean**’), ‘**median**’ and **mode**.

The **range** is the difference between the lowest and highest values. For example:

* The lowest value is 5
* The highest value is 11

Very simple but the result can be very misleading if there is an extraordinarily high or low value in the data set compared to the rest.

The **average** or **mean** value is defined as the number that measures the central tendency of a given set of numbers. You calculate this by adding up all the numbers in the data set and dividing this answer by the number of items in the data set. For example, using the numbers above:

The **median** is the middle value of a data set. To find the median, list the values of the data set in numerical order and identify which value appears in the middle of the list. For example, again using the data set above:

|  |
| --- |
|  |

The **mode** is the value that occurs the most and there can be more than one. For example, using the data set above it can be seen that the values that occur the most is **6** and **8** with two of each so these are the mode values.

**Useful formulae**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **VOLTAGE** | = | I.R | | = | P/I | | = | √P.R |
| **CURRENT** | = | V/R | | = | P/V | | = | √P/R |
| **RESISTANCE** | = | V/I | | = | V2/P | | = | P/I2 |
| **POWER** | = | V.I | | = | I2.R | | = | V2/R |
| **RESISTORS** | In series: | |  | | | etc | | | |
|  | in parallel: | |  | | | etc | | | |

**KIRCHHOFF’S LAWS**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Voltage: | VT = V1 + V2 + V3 + etc | | | | | | | | in a Series circuit | | | | | |
| Current: | IT = I1 + I2 + I3 + etc | | | | | | | | in a Parallel circuit | | | | | |
| **RESISTIVITY** |  | | | | R ∝ L | | | | | R ∝ 1/a | | |
| **CHARGE** (Quantity) | | Q = It or | | | | | | | Q = VC | | | | | |
| **CAPACITORS** | in parallel: | | |  | | | | | | | etc | | | |
|  | in series: | | |  | | | | | | | etc | | | |
| **ELECTRO- MAGNETISM** | Φ | | | | | | = | B.A | | | | B ∝ H | | |
|  | mmf | | | | | | = | N x I | | | |  | | |
|  | H | | | | | | = |  | | | |  | | |
| **INDUCED EMF** | E | | | | | | = |  | | | | | |  |
|  | E | | | | | | = |  | | | | | |  |
|  | E | | | | | | = |  | | | | | |  |
| **FORCE ON A CONDUCTOR** | | | F | | | = | |  | | | | | |  |
| **ENERGY** | in a magnetic field: | | | | | | | | ½.L.I2 Joules | | | | | |
| **STORED** | in a capacitor: | | | | | | | | ½.C.V2 Joules | | | | | |
| **EFFICIENCY** | Efficiency | | | | | | = |  | | | | | | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **MECHANICS** | Force | | = | Mass x Newtons |
|  | Torque | | = | Force x Distance |
|  | Work Done (WD) | | = | Force x Distance |
|  | Energy | | = | Joules |
|  | Joules | | = | Watts x Seconds |
|  | Joules | | = | Newton Metre |
| **PYTHAGORAS** | | Z | = |  |
| **POWER FACTOR** | | p.f. | = |  |
| **POWER FACTOR** | | p.f. | = |  |